Dissection of rhombic solids with octahedral symmetry to Archimedean solids, Part 2

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In [5] we found "best approximate" by "rhombic solids" for certain Platonic and Archimedean solids with octahedral symmetry. In this paper we treat the remaining solids: truncated cube, rhombicubocahedron and truncated cuboctahedron. For this purpose we extend the notion of rhombic solid so, that it is also possible to add to them 1/4 and 1/8 of the rhombic dodecahedron and 1/6 of the cube. These last solids can be dissected to cubes.
The authors of [1, pg. 331] produced the table of the Dehn invariants for the non-snub unit edge Archimedean polyhedra. We have considered the icosahedral part in [7].
Results from both papers show, that each best aproximate has a surplus or a deficit
measured in tetrahedrons (T): -2, -1, 0, 1, 2. Actualy 1/4 of the tetrahedron appears as a piece.
The table of Dehn invariant for the non-snub unit edge Archimedean polyhedra (octahedral part) [1]:

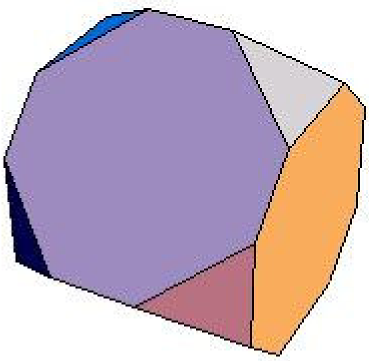
Tetrahedron	-12(3)2	
Truncated tetrahedron	12(3)2	
Cube	0	
Truncated cube	-24(3)2	
Octahedron	24(3)2	
Truncated octahedron	0	
Rhombicuboctahedron	-24(3)2	???
Cuboctahedron	-24(3)2	
Truncated cuboctahedron	0	

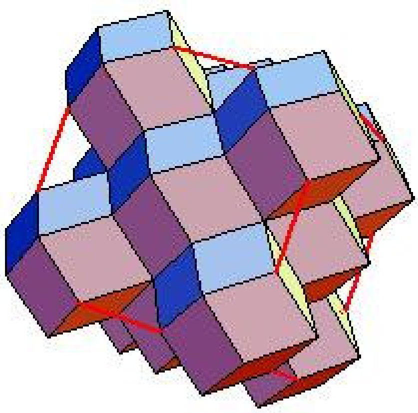
Along with the above table, we produced the following one:

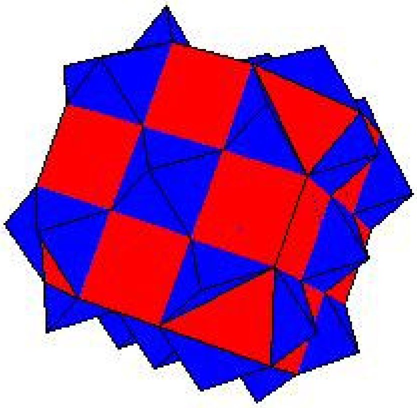
Tetrahedron	1
Truncated tetrahedron	-1
Cube	0
Truncated cube	2
Octahedron	-2
Truncated octahedron	0
Rhombicuboctahedron	-2
Cuboctahedron	2
Truncated cuboctahedron	0

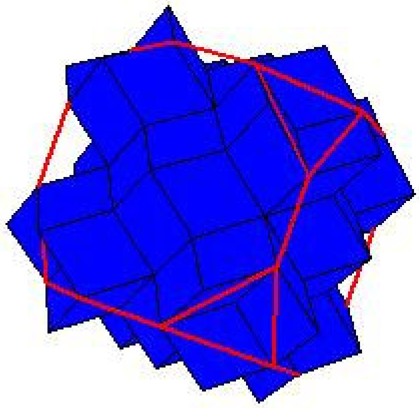
For instance: the tetrahedron has a surplus of 1 tetrahedron relative to empty polyhedron,
the octahedron has deficit of 2 tetrahedra relative to rhombic dodecahedron.

Our truncated cube, rhombicuboctahedron and truncated cuboctahedron are not Archimedean, but can be enlarged to Archimedean by addition of some prisms, so the results in the table are valid for Archimedean solids as well.

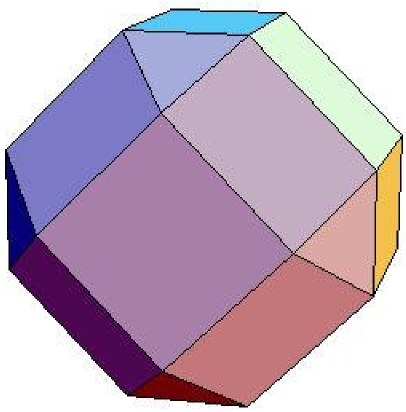


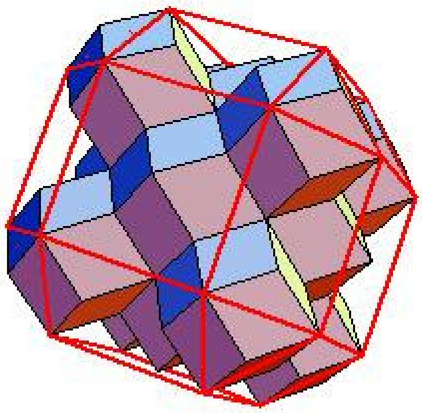




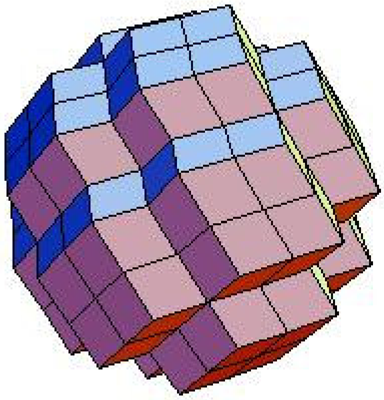


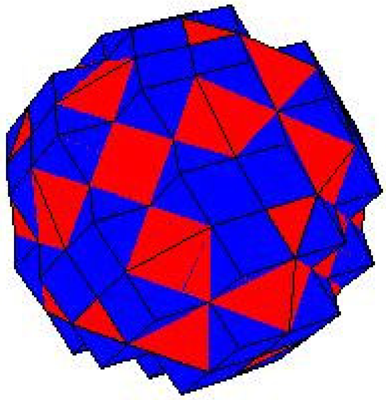
Observing triangular faces of the truncated cube we see a surplus of 1/4 of the tetrahedron at each face. At octagonal faces we observe a surplus of four eights and one half of the cube and a deficit of four halves of the cube. This gives a deficit of one half of the cube for each face. This means that we should add 3 small cubes (or 3/2 of rhombic dodecahedron) to obtain the best approximate of truncated cube.

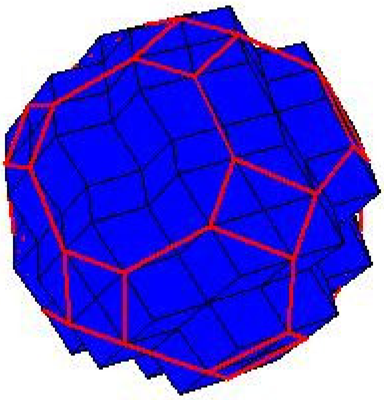


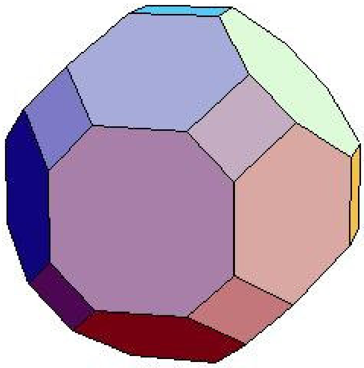


Observing triangular faces of the rhombic cuboctahedron we observe a deficit of 1/4 of the tetrahedron at each. Each rectangular face lacks two quarters of rhombic dodecahedron and square face lacks four eights of the rhombic dodecahedron. So to obtain the best approximate these parts of rhombic dodecahedron should be added.









Observing hexagonal faces oh truncated cuboctahedron we observe surplus and deficit of three triangular pyramids (1/4 of tetrahedron) On octagonal face we have still a deficit of a half of the cube, so we shoud add these halves to get the best approximate.

References

[1] J. H. Conway, C. Radin, and L. Sadun, On angles whose squared trigonometric functions are rational, Discrete & Computational Geometry, 22 (1999), pages 321-332.
[2] G. N. Frederickson, Dissections: Plane & Fancy, Cambridge U. Press, 1997.
[3] G. N. Frederickson, Hinged Dissections: Swinging & Twisting, Cambridge U. Press, 2002.
[4] R. Williams, The Geometrical Foundation Of Natural Structure, Dover 1972
[5] I. Hafner, T. Zitko, Dissection of rhombic 36-hedron to a tetrahedron and a truncated tetrahedron
[6] I. Hafner, Dissection of rhombic solids with octahedral symmetry to Archimedean solids, Part 1
[7] I. Hafner, Solution of Conway-Radin-Sadun Problem, Summary of Results.